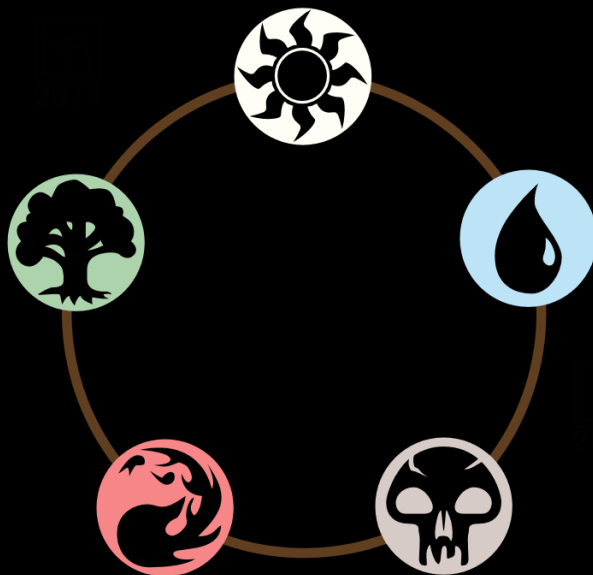


# MAGIC Old School

The Gathering®



## An Introduction to the Hypergeometric Distribution for Magic Players

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How often will you draw a specific spell or a land in your opening hand? How often will you draw multiples? All of these questions can be answered with the help of a hypergeometric distribution. By using a hypergeometric calculator, getting the percentages is easy, and in this article I'll explain how.

## Using a Hypergeometric Calculator

The hypergeometric distribution can describe the likelihood of any number of successes when drawing from a deck of Magic cards. It takes into account the fact that each draw decreases the size of your library by one, and therefore the probability of success changes on each draw.

To be able to apply the hypergeometric distribution, it is essential that your deck can be classified into two mutually exclusive categories. A probability textbook might talk about success/failure, but in a Magic context it could be likelihood of drawing a creature/non-creature, or likelihood of drawing a land.

The easiest way to get some quick probabilities is by using an *online hypergeometric calculator*.

I like the one from [Stattrek.com](#), which looks like the picture below:

*(click on the link above to access the online calculator and to type in your numbers)*

- Enter a value in each of the first four text boxes (the unshaded boxes).
- Click the **Calculate** button.

Population size

Number of successes in population

Sample size

Number of successes in sample (x)

Hypergeometric Probability:  $P(X = x)$

Cumulative Probability:  $P(X < x)$

Cumulative Probability:  $P(X \leq x)$

Cumulative Probability:  $P(X > x)$

Cumulative Probability:  $P(X \geq x)$

**Calculate**

To use it, we need to plug four numbers into the calculator:

- **Population size.** In a Magic context, this is the number of cards in the deck when the card draw experiment starts. For example, if we have a 60-card Modern deck and we're calculating the probability of drawing something in our opening hand, then this would be 60.
- **Number of successes in population.** For Magic, this is the number of functionally identical copies of cards in our deck that we want to draw. For example, if we're calculating the probability of drawing one of our four [Hardened Scales](#) magic cards in our opening hand, then this would be 4.
- **Sample size.** The number of cards we're drawing. For example, if we are interested in our opening hand, then this would be 7.
- **Number of successes in sample.** How many copies we want to draw. For example, if care about having one **Hardened Scales** card in our opening hand, then this would be 1.

If we plug in the numbers, then the output is a set of probabilities. Note that to convert them into a percentage, you multiply by 100%. So a probability of 0.399 is the same as 39.9%.

Population size	<input type="text" value="60"/>	<b>Cards in deck</b>
Number of successes in population	<input type="text" value="4"/>	<b>Hardened Scales in deck</b>
Sample size	<input type="text" value="7"/>	<b>Cards in opening hand</b>
Number of successes in sample (x)	<input type="text" value="1"/>	<b>Hardened Scales in opening hand</b>
Hypergeometric Probability: $P(X = 1)$	<input type="text" value="0.33628021"/>	<b>Draw exactly 1 Hardened Scales</b>
Cumulative Probability: $P(X < 1)$	<input type="text" value="0.600500374"/>	<b>Draw 0 Hardened Scales</b>
Cumulative Probability: $P(X \leq 1)$	<input type="text" value="0.936780584"/>	<b>Draw 0 or 1 Hardened Scales</b>
Cumulative Probability: $P(X > 1)$	<input type="text" value="0.063219416"/>	<b>Draw 2, 3, or 4 Hardened Scales</b>
Cumulative Probability: $P(X \geq 1)$	<input type="text" value="0.399499626"/>	<b>Draw 1, 2, 3, or 4 Hardened Scales</b>

The way to interpret this output is as follows:

- **Hypergeometric Probability:  $P(X = 1)$ .** The probability of drawing exactly 1 Hardened Scales. This is usually not what we're interested in. Instead, the probability of drawing at least one Hardened Scales tends to be more relevant.
- **Cumulative Probability:  $P(X < 1)$ .** The probability of drawing 0 copies. This would constitute the *likelihood of a failure*, i.e., a 7-card opening hand without Hardened Scales or another example could be without a land or colored mana.
- **Cumulative Probability:  $P(X \leq 1)$ .** The probability of drawing 0 or 1 copies. This is generally not interesting to us.
- **Cumulative Probability:  $P(X > 1)$ .** The probability of drawing 2 or more copies. This is generally not interesting to us.
- **Cumulative Probability:  $P(X \geq 1)$ .** The probability of drawing at least 1 copy. Usually, this is the number that we are interested in as this would often be the *likelihood of success*. So, the probability to draw at least one of your four Hardened Scales in your opening hand from a 60-card deck is 39.9%.

An alternative to this online hypergeometric calculator is the use of Excel or Google Sheets. As an example, on how to use those tools, the “failure” probability of drawing exactly 0 Hardened Scales in your opening hand when you’re playing a 60-card deck with 4 copies is calculated as follows:

- In Excel: =HYPGEOM.DIST(0,7,4,60,FALSE)
- In Google Sheets: =HYPGEOMDIST(0,7,4,60)

## Example: Risk of being mana screwed

Imagine you are building a 3 colored deck with 60 cards and have included following mana sources in the mainboard:

- 1 Library of Alexandria
- 1 Strip Mine
- 4 Mishra’s factory
- 4 City of Brass
- 10 Dual Lands (8 of these can produce blue mana)
- 3 On color Moxes

### Potential interesting Questions:

1) *What is the likelihood of starting with at least two mana sources in your seven-card starting hand?*

This would look like this:

- Enter a value in each of the first four text boxes (the unshaded boxes).
- Click the **Calculate** button.

Population size	<input type="text" value="60"/>
Number of successes in population	<input type="text" value="24"/>
Sample size	<input type="text" value="7"/>
Number of successes in sample (x)	<input type="text" value="2"/>
Hypergeometric Probability: $P(X = 2)$	<input type="text" value="0.269414624"/>
Cumulative Probability: $P(X < 2)$	<input type="text" value="0.14265588"/>
Cumulative Probability: $P(X \leq 2)$	<input type="text" value="0.412070504"/>
Cumulative Probability: $P(X > 2)$	<input type="text" value="0.587929496"/>
Cumulative Probability: $P(X \geq 2)$	<input type="text" value="0.85734412"/>

The likelihood of you to draw at least 2 mana sources in your opening hand is 85,7%

2) What is the likelihood of starting with at least 1 or 2 colored mana sources in your seven-card starting hand?

This would look like this:

- Enter a value in each of the first four text boxes (the unshaded boxes).
- Click the **Calculate** button.

Population size	<input type="text" value="60"/>
Number of successes in population	<input type="text" value="17"/>
Sample size	<input type="text" value="7"/>
Number of successes in sample (x)	<input type="text" value="1"/>
Hypergeometric Probability: $P(X = 1)$	<input type="text" value="0.268352825"/>
Cumulative Probability: $P(X < 1)$	<input type="text" value="0.083437433"/>
Cumulative Probability: $P(X \leq 1)$	<input type="text" value="0.351790258"/>
Cumulative Probability: $P(X > 1)$	<input type="text" value="0.648209742"/>
Cumulative Probability: $P(X \geq 1)$	<input type="text" value="0.916562567"/>

- Enter a value in each of the first four text boxes (the unshaded boxes).
- Click the **Calculate** button.

Population size	<input type="text" value="60"/>
Number of successes in population	<input type="text" value="17"/>
Sample size	<input type="text" value="7"/>
Number of successes in sample (x)	<input type="text" value="2"/>
Hypergeometric Probability: $P(X = 2)$	<input type="text" value="0.338971989"/>
Cumulative Probability: $P(X < 2)$	<input type="text" value="0.351790258"/>
Cumulative Probability: $P(X \leq 2)$	<input type="text" value="0.690762247"/>
Cumulative Probability: $P(X > 2)$	<input type="text" value="0.309237753"/>
Cumulative Probability: $P(X \geq 2)$	<input type="text" value="0.648209742"/>

The likelihood of you to draw at least 1 colored mana sources is 91,7%% and the likelihood of drawing at least 2 colored mana sources in your starting hand is 64,8%.

This would be an acceptable success if your cards in your deck primarily require one colored mana – but would be an unacceptable likelihood if your cards in your deck primarily require two colored mana

3) What is the likelihood of starting with at least 1 blue\_mana source in your seven-card starting hand?

This would look like this:

- Enter a value in each of the first four text boxes (the unshaded boxes).
- Click the **Calculate** button.

Population size	<input style="width: 100%;" type="text" value="60"/>
Number of successes in population	<input style="width: 100%;" type="text" value="13"/>
Sample size	<input style="width: 100%;" type="text" value="7"/>
Number of successes in sample (x)	<input style="width: 100%;" type="text" value="1"/>
Hypergeometric Probability: $P(X = 1)$	<input style="width: 100%;" type="text" value="0.361434355"/>
Cumulative Probability: $P(X < 1)$	<input style="width: 100%;" type="text" value="0.16284405"/>
Cumulative Probability: $P(X \leq 1)$	<input style="width: 100%;" type="text" value="0.524278405"/>
Cumulative Probability: $P(X > 1)$	<input style="width: 100%;" type="text" value="0.475721595"/>
Cumulative Probability: $P(X \geq 1)$	<input style="width: 100%;" type="text" value="0.83715595"/>

The likelihood for you to draw at least 1 blue mana source is 83,7%%. (counting 4 city of brass, blue mox and 8 dual lands that produce blue in the mainboard). If blue is critical for the deck to operate smoothly – then it should be considered adding a few more blue mana sources to bring this to 90%.

From the calculation below goal seeking a 90% likelihood - this would require 3 extra blue mana sources:

- Enter a value in each of the first four text boxes (the unshaded boxes).
- Click the **Calculate** button.

Population size	<input type="text" value="60"/>
Number of successes in population	<input type="text" value="16"/> +3
Sample size	<input type="text" value="7"/>
Number of successes in sample (x)	<input type="text" value="1"/>
Hypergeometric Probability: $P(X = 1)$	<input type="text" value="0.292446422"/>
Cumulative Probability: $P(X < 1)$	<input type="text" value="0.099222893"/>
Cumulative Probability: $P(X \leq 1)$	<input type="text" value="0.391669316"/>
Cumulative Probability: $P(X > 1)$	<input type="text" value="0.608330684"/>
Cumulative Probability: $P(X \geq 1)$	<input type="text" value="0.900777107"/>

## What is an Acceptable Percentage?

When building decks, you can use hypergeometric probabilities to get an idea of success and failure rates, but what is an acceptable percentage, and how should that translate to how many copies of a certain type of card to play? The answer depends on a lot of factors, most importantly the impact of things going right or wrong. But I can give some general guidelines, based on experience and successful tournament decks:

- 90% is an acceptable success rate when the failure case is a disaster. For example, not having the right colored source of mana to play your spells, not hitting a land drop in turn 2 and you really don't want these things to happen more than once every ten games.
- 80% is an acceptable success rate when the failure case is not as bad. I might even accept 70% consistency, depending on the card availability in the format. In the end, it's all a trade-off.

## Conclusion

Using the hypergeometric distribution to analyze the consistency of your deck or to figure out how many copies of a certain card to play is pretty easy, and I hope that this article was helpful in explaining how.